LEARNING STATISTICS BASED UPON MULTIPLE THEORIES OF PROBABILITY

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Introduction:
Probabilistic inference is the foundation of statistics. Hypothesis testing - a fundamental process that underlies the science of statistics – is based on the probabilistic inference of obtaining a given statistic in the long run, given that the null hypothesis is true. Despite the utility of learning diverse perspectives and theories on probability, many students learn the subject of probability within a single and unified framework (the frequency approach). As a result, statistics learners often learn to blindly follow mechanistic principles (e.g. alpha ≤ 0.05). Students' failure to learn that probability is a complex, multitheoretical subject often interferes with their conceptual comprehension of the wider subject of statistics and their openness to different interpretations.

Statistics is built upon multiple theories of probability:
Probability theories include the classical theory, the frequency approach, the Bayesian model (Berry, 1996), the notion of propensity (Gillies, 2012), and many others. The multitudinous, occasionally contradictory, nature of such theories, compounded with psychological fallacies (i.e. the 'above-average fallacy', the 'unreliability of eye-witness testimony', and the 'conjunction fallacy,') hinder students from making correct probabilistic inferences.

Conclusion:
Although statistics is based on philosophy, logical reasoning (Onwuegbuzie & Wilson, 2003), and multitheoretical perspectives on probability, students in introductory statistics classes are traditionally taught the subject of probability within a uniform, frequency approach (Galavotti, 2005). Rather than being taught in a straightforward, singular manner, statistics should be taught within the context of relevant factors (i.e. independence of chance, regression toward the mean, Bayesian conditional probability, propensity, and direct inference), and with the aid of real-world examples. Based on their teaching experience, these authors believe that this philosophical and psychological approach would help students overcome common fallacies and difficulties in learning the subject of probabilistic inference.

EXAMPLE QUESTION 1: IF A STUDENT EARNS THREE 'A'S IN A ROW, WHAT IS HIS OR HER PROBABILITY OF EARNING AN 'A' ON HIS OR HER NEXT TEST?

Approach 1: Law of independence and the problem of induction: According to this view, every trial (test) is independent, and previous performance has nothing to do with the outcomes of subsequent events. A common sense approach may indicate that a student's past successes indicates that it is likely that he or she will do well in the future. David Humes (1777/1912) and Nassim Taleb (2008) challenged this logic, questioning whether the future necessarily resembles the past. Given the view that every single trial (test) is independent, the fact that this student earned three 'A's on his or her first test does not affect the probability of his or her earning an 'A' on his or her fourth test.

P(Prior probability of getting an 'A' on test 4) = P(Prior probability of getting an 'A' on test 4)

Figure 1. Probability of getting an 'A' on test 4, using the 'law of independence' approach

Approach 2: Regression towards the mean: 'Regression' means 'going back' or 'going down' - whatever goes up will eventually go down. According to Francis Galton (1886) and Daniel Kahneman (2011), after a series of events in which a student performs well, the probability of this student performing poorly will Increase. Given the view that luck and other factors outside the control of the student may have played a role in this students' prior successes, and that some uncontrollable factors may not be repeated in the next situation, the probability of this student's earning an 'A' on his or her fourth decreases, given his or her first four successes.

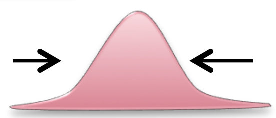


Figure 2. Regression toward the mean

P(Prior probability of getting an 'A' on test 4) > P(Posterior probability of getting an 'A' on test 4)

Figure 3. Probability of getting an 'A' on test 4, using the 'regression toward the mean' approach

Conditional probability: According to a Bayesian perspective, a student's past successes can be used to establish background information and prior probabilities. In order to establish posterior probabilities, more specific information (about this student and the test itself) is needed. For example, given the conditions that a student continues to work hard and the difficulty level of the upcoming test is similar to the previous tests, the probability that she would earn another "A: is ____. Therefore, given the view that this student's prior successes are indicative of his or her superior ability, and given that he or she continues to spend the same time and effort in preparing for test 4 as he or she spent on preparing for tests 1-3, his or her probability of earning an 'A' on his or her fourth test increases, given his or her first three successes.

P(Prior probability of getting an 'A' on test 4) < P(Prior probability of getting an 'A' on test 4)

Figure 4. Probability of getting an 'A' on test 4, using the 'conditional probability' approach

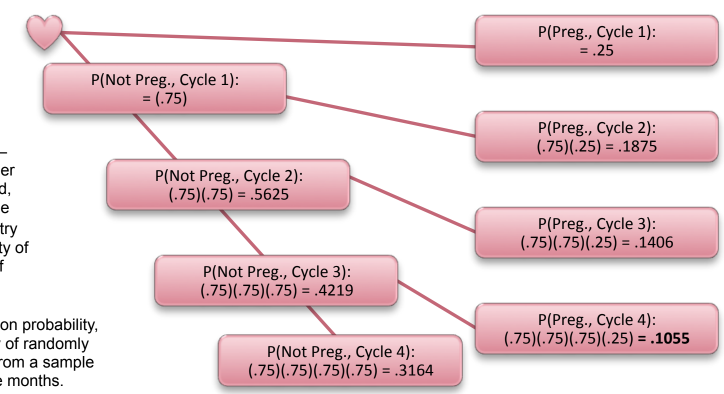
EXAMPLE QUESTION 2: IN A SAMPLE OF HEALTHY, 24-YEAR-OLD FEMALES, WHAT IS THE PROBABILITY OF SELECTING ONE WHO CONCEIVES IN HER FOURTH – VS. HER FIRST - CYCLE OF PREGNANCY?

Approach 1, Frequency Probability: According to this view, a female is treated as a member of a super-population: healthy females. This super-population's average monthly probability of getting pregnant is around 25% for females aged 20-30, and 10% for women aged 35+ (George & Kamath, 2010); these probabilities are derived from a statistical law governing the given population. Since it is assumed that every event of this set is equi-probable, the probability of selecting a healthy woman who has conceived in her fourth cycle of pregnancy (from a sample of females who have already tried to conceive for three consecutive cycles), is the same as the probability of selecting a healthy woman who has conceived in her first cycle of pregnancy.

P(Pregnant, Cycle 1): 0.25 = P(Pregnant, Cycle 2): 0.25 = P(Pregnant, Cycle 3): 0.25 = P(Pregnant, Cycle 4): **0.25**

Figure 5. Probability of randomly selecting a healthy woman who has conceived in her fourth cycle of pregnancy, using a 'frequency probability' approach

Figure 6. Probability of randomly selecting a female who has conceived in her fourth cycle of pregnancy, using a 'conditional probability' approach



Approach 2, Conditional Probability:
This approach takes into account measures of uncertainty – i.e. age, regularity of menstrual cycle, woman's BMI, whether or not woman was taking prenatal vitamins and/or folic acid, whether or not this was a first-time pregnancy, fallopian tube functionality, quality of sperm, and average income of country woman was from. According to this approach, the probability of selecting a 24-year-old who conceives in her fourth cycle of attempting, will range widely.

Using only prior failures to conceive as predictors of selection probability, *Figure 6* displays a calculation of the conditional probability of randomly selecting a female who has conceived in her fourth cycle, from a sample of females who have been attempting for three consecutive months.

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